1 - 9 General solution

Find a real general solution of the following systems.

1. $y_1' = y_1 + y_2$

$y_2' = 3 y_1 - y_2$

```
ClearAll["Global`*"]
```

Mathematica solves the system, but to knock the solutions into a framework which can be directly compared with the text answer, some wrangling, rearranging, and substituting must be done.

```
rit = {y1'[t] == y1[t] + y2[t], y2'[t] == 3 y1[t] - y2[t]}
git = DSolve[rit, {y1, y2}, t]
```

```
{y1′[t] == y1[t] + y2[t], y2′[t] == 3 y1[t] - y2[t]}
```

$$\{\{y1 \to \text{Function}[\{t\}, \frac{1}{4} e^{-2t} (1 + 3 e^{4t}) C[1] + \frac{1}{4} e^{-2t} (-1 + e^{4t}) C[2]],$$
$$y2 \to \text{Function}[\{t\}, \frac{3}{4} e^{-2t} (-1 + e^{4t}) C[1] + \frac{1}{4} e^{-2t} (3 + e^{4t}) C[2]]\}\}$$

```
fit = Expand[git[[1, 1, 2, 2]]]
```

$$\frac{1}{4} e^{-2t} C[1] + \frac{3}{4} e^{2t} C[1] - \frac{1}{4} e^{-2t} C[2] + \frac{1}{4} e^{2t} C[2]$$

```
vit = Expand[4 fit]
```

$$e^{-2t} C[1] + 3 e^{2t} C[1] - e^{-2t} C[2] + e^{2t} C[2]$$

```
bit = Collect[vit, e^-2 t]
```

$$e^{-2t} (C[1] - C[2]) + e^{2t} (3 C[1] + C[2])$$

Having reconciled the form of the constants of integration, a recognizable variant emerges.

```
mit = bit /. {(C[1] - C[2]) → c1, (3 C[1] + C[2]) → c2}
```

$$c1\, e^{-2t} + c2\, e^{2t}$$

```
wit = Expand[git[[1, 2, 2, 2]]]
```

$$-\frac{3}{4} e^{-2t} C[1] + \frac{3}{4} e^{2t} C[1] + \frac{3}{4} e^{-2t} C[2] + \frac{1}{4} e^{2t} C[2]$$

```
pit = Expand[4 wit]
```

$$-3 e^{-2t} C[1] + 3 e^{2t} C[1] + 3 e^{-2t} C[2] + e^{2t} C[2]$$

```
sit = Collect[pit, ⅇ^(-2 t)]
```
$e^{2t}(3 C[1] + C[2]) + e^{-2t}(-3 C[1] + 3 C[2])$

```
kit = sit /. (-3 C[1] + 3 C[2]) → (-3 (C[1] - C[2]))
```
$-3 e^{-2t}(C[1] - C[2]) + e^{2t}(3 C[1] + C[2])$

```
lit = kit /. {(C[1] - C[2]) → c1, (3 C[1] + C[2]) → c2}
```
$-3\, c1\, e^{-2t} + c2\, e^{2t}$

1. Above: The top green cell ‘mit’ is y1, the bottom green cell ‘lit’ is y2. They both match the text expressions, even to the constants. Care was taken to make sure equal constant substitutions were made in both cases (yellow).

3. $y_1' = y_1 + 2 y_2$
$y_2' = y_1 + 2 y_2$

```
ClearAll["Global`*"]
```

```
nar = {y1'[t] == y1[t] + 2 y2[t], y2'[t] == 1/2 y1[t] + y2[t]}
bar = DSolve[nar, {y1, y2}, t]
```
$\{y1'[t] == y1[t] + 2\, y2[t],\ y2'[t] == \frac{y1[t]}{2} + y2[t]\}$

$\{\{y1 \to \text{Function}[\{t\}, \frac{1}{2}(1 + e^{2t}) C[1] + (-1 + e^{2t}) C[2]],$
$y2 \to \text{Function}[\{t\}, \frac{1}{4}(-1 + e^{2t}) C[1] + \frac{1}{2}(1 + e^{2t}) C[2]]\}\}$

```
mar = Expand[bar[[1, 1, 2, 2]]]
```
$\frac{C[1]}{2} + \frac{1}{2} e^{2t} C[1] - C[2] + e^{2t} C[2]$

```
uar = Expand[2 mar]
```
$C[1] + e^{2t} C[1] - 2 C[2] + 2 e^{2t} C[2]$

```
sar = uar /. ( C[1] - 2 C[2]) → ( c2)
```
$c2 + e^{2t} C[1] + 2 e^{2t} C[2]$

```
tar = Collect[sar, ⅇ^(2 t)]
```
$c2 + e^{2t}(C[1] + 2 C[2])$

```
var = tar /. (C[1] + 2 C[2]) → c1
```

$c2 + c1\, e^{2t}$

```
jar = Expand[2 var]
```

$2\, c2 + 2\, c1\, e^{2t}$

```
par = Expand[bar[[1, 2, 2, 2]]]
```

$-\frac{C[1]}{4} + \frac{1}{4} e^{2t} C[1] + \frac{C[2]}{2} + \frac{1}{2} e^{2t} C[2]$

```
har = Expand[4 par]
```

$-C[1] + e^{2t} C[1] + 2\, C[2] + 2\, e^{2t} C[2]$

```
dar = har /. ( -C[1] + 2 C[2]) → (- c2)
```

$-c2 + e^{2t} C[1] + 2\, e^{2t} C[2]$

```
qar = Collect[dar, e^(2 t)]
```

$-c2 + e^{2t} (C[1] + 2\, C[2])$

```
xar = qar /. (C[1] + 2 C[2]) → c1
```

$-c2 + c1\, e^{2t}$

1. Above: The functions ‘jar’ and ‘xar’, (upper and lower green cells respectively), are y1 and y2, and match the text answer. Note that in assembling the functions, each was multiplied by 4. However, care was taken so that the proportions and signs of the constants match those of the text.

5. $y_1{}' = 2\, y_1 + 5\, y_2$
$y_2{}' = 5\, y_1 + 12.5\, y_2$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == 2 y1[t] + 5 y2[t], y2'[t] == 5 y1[t] + 12.5 y2[t]}
e2 = DSolve[e1, {y1, y2}, t]
```

{y1′[t] == 2 y1[t] + 5 y2[t], y2′[t] == 5 y1[t] + 12.5 y2[t]}

$\{\{y1 \to \text{Function}[\{t\}, 0.137931\, e^{-2.22045\times10^{-16}\, t} (6.25 + 1.\, e^{14.5\, t})\, C[1] + 0.344828\, e^{-2.22045\times10^{-16}\, t} (-1. + 1.\, e^{14.5\, t})\, C[2]],$
$y2 \to \text{Function}[\{t\}, 0.344828\, e^{-2.22045\times10^{-16}\, t} (-1. + 1.\, e^{14.5\, t})\, C[1] + 0.862069\, e^{-2.22045\times10^{-16}\, t} (0.16 + 1.\, e^{14.5\, t})\, C[2]]\}\}$

```
e3 = e2[[1, 1, 2, 2]]
```

$$0.137931\, e^{-2.22045\times10^{-16}\, t} \left(6.25 + 1.\, e^{14.5\, t}\right) C[1] + 0.344828\, e^{-2.22045\times10^{-16}\, t} \left(-1. + 1.\, e^{14.5\, t}\right) C[2]$$

```
e16 = e3 /. {C[1] → C1, C[2] → C2}
```

$$0.344828\, C2\, e^{-2.22045\times10^{-16}\, t} \left(-1. + 1.\, e^{14.5\, t}\right) + 0.137931\, C1\, e^{-2.22045\times10^{-16}\, t} \left(6.25 + 1.\, e^{14.5\, t}\right)$$

```
e4 = Chop[e16, 10^-15]
```

$$0.344828\, C2 \left(-1. + 1.\, e^{14.5\, t}\right) + 0.137931\, C1 \left(6.25 + 1.\, e^{14.5\, t}\right)$$

```
e5 = Expand[e4]
```

$$0.862069\, C1 - 0.344828\, C2 + 0.137931\, C1\, e^{14.5\, t} + 0.344828\, C2\, e^{14.5\, t}$$

```
e6 = Collect[e5, e^(14.5 t)]
```

$$0.862069\, C1 - 0.344828\, C2 + (0.137931\, C1 + 0.344828\, C2)\, e^{14.5\, t}$$

```
e7 = e6 /. (0.13793103448275862` C1 + 0.3448275862068965` C2) → 2 c2
```

$$0.862069\, C1 - 0.344828\, C2 + 2\, c2\, e^{14.5\, t}$$

```
e8 = e7 /. (0.8620689655172413` C1 - 0.3448275862068966` C2) → 5 c1
```

$$5\, c1 + 2\, c2\, e^{14.5\, t}$$

```
Solve[0.13793103448275862` + 0.3448275862068965` == 2 c21, c21]
```

```
{{c21 → 0.241379}}
```

```
Solve[0.8620689655172413` - 0.3448275862068966` == 5 c11, c11]
```

```
{{c11 → 0.103448}}
```

```
e10 = e2[[1, 2, 2, 2]]
```

$$0.344828\, e^{-2.22045\times10^{-16}\, t} \left(-1. + 1.\, e^{14.5\, t}\right) C[1] + 0.862069\, e^{-2.22045\times10^{-16}\, t} \left(0.16 + 1.\, e^{14.5\, t}\right) C[2]$$

```
e17 = e10 /. {C[1] → C1, C[2] → C2}
```

$$0.344828\, C1\, e^{-2.22045\times10^{-16}\, t} \left(-1. + 1.\, e^{14.5\, t}\right) + 0.862069\, C2\, e^{-2.22045\times10^{-16}\, t} \left(0.16 + 1.\, e^{14.5\, t}\right)$$

```
e11 = Chop[e17, 10^-15]
```

$$0.344828\, C1 \left(-1. + 1.\, e^{14.5\, t}\right) + 0.862069\, C2 \left(0.16 + 1.\, e^{14.5\, t}\right)$$

```
e12 = Expand[e11]
```

$-0.344828\,C1 + 0.137931\,C2 + 0.344828\,C1\,e^{14.5\,t} + 0.862069\,C2\,e^{14.5\,t}$

```
e13 = Collect[e12, e^(14.5 t)]
```

$-0.344828\,C1 + 0.137931\,C2 + (0.344828\,C1 + 0.862069\,C2)\,e^{14.5\,t}$

```
e14 = e13 /. (0.3448275862068966` C1 + 0.8620689655172414` C2) → 5 c2
```

$-0.344828\,C1 + 0.137931\,C2 + 5\,c2\,e^{14.5\,t}$

```
Solve[0.3448275862068966` + 0.8620689655172414` == 5 c22, c22]
```

{{c22 → 0.241379}}

```
e15 = e14 /. (-0.3448275862068965` C1 + 0.13793103448275862` C2) → (-2 c1)
```

$-2\,c1 + 5\,c2\,e^{14.5\,t}$

```
Solve[-0.3448275862068965` + 0.13793103448275862` == -2 c21, c21]
```

{{c21 → 0.103448}}

1. Above: y1 is given by e8; y2 is given by e15. These expressions match the text answers. Green cells are for function formulas, yellow cells for sites of assignment of values of constants, pink cells for constant value verification. The equality of c11,c12; and c21,c22 shows that due consideration was given to preserving the values and proportions of the constants. (In calculating the numerical value of constants for comparison, the values of Mathematica's constants was taken as all 1.)

7. $y_1' = y_2$
$y_2' = -y_1 + y_3$
$y_3' = -y_2$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == y2[t], y2'[t] == -y1[t] + y3[t], y3'[t] == -y2[t]}
e2 = DSolve[e1, {y1, y2, y3}, t]
```

{y1′[t] == y2[t], y2′[t] == -y1[t] + y3[t], y3′[t] == -y2[t]}

$$\left\{\left\{y1 \to \text{Function}\left[\{t\}, \frac{1}{2} C[3] \left(1 - \text{Cos}\left[\sqrt{2}\, t\right]\right) + \frac{1}{2} C[1] \left(1 + \text{Cos}\left[\sqrt{2}\, t\right]\right) + \frac{C[2]\, \text{Sin}\left[\sqrt{2}\, t\right]}{\sqrt{2}}\right],\right.\right.$$

$$y2 \to \text{Function}\left[\{t\}, C[2]\, \text{Cos}\left[\sqrt{2}\, t\right] - \frac{C[1]\, \text{Sin}\left[\sqrt{2}\, t\right]}{\sqrt{2}} + \frac{C[3]\, \text{Sin}\left[\sqrt{2}\, t\right]}{\sqrt{2}}\right],$$

$$\left.\left. y3 \to \text{Function}\left[\{t\}, \frac{1}{2} C[1] \left(1 - \text{Cos}\left[\sqrt{2}\, t\right]\right) + \frac{1}{2} C[3] \left(1 + \text{Cos}\left[\sqrt{2}\, t\right]\right) - \frac{C[2]\, \text{Sin}\left[\sqrt{2}\, t\right]}{\sqrt{2}}\right]\right\}\right\}$$

```
e3 = e2[[1, 1, 2, 2]]
```

$$\frac{1}{2} C[3] \left(1 - \text{Cos}\left[\sqrt{2}\, t\right]\right) + \frac{1}{2} C[1] \left(1 + \text{Cos}\left[\sqrt{2}\, t\right]\right) + \frac{C[2]\, \text{Sin}\left[\sqrt{2}\, t\right]}{\sqrt{2}}$$

```
e4 = Expand[ 2 e3]
```

$$C[1] + C[3] + C[1]\, \text{Cos}\left[\sqrt{2}\, t\right] - C[3]\, \text{Cos}\left[\sqrt{2}\, t\right] + \sqrt{2}\, C[2]\, \text{Sin}\left[\sqrt{2}\, t\right]$$

$$\text{e5} = \text{Collect}\left[\text{e4}, \text{Cos}\left[\sqrt{2}\, t\right]\right]$$

$$C[1] + C[3] + (C[1] - C[3])\, \text{Cos}\left[\sqrt{2}\, t\right] + \sqrt{2}\, C[2]\, \text{Sin}\left[\sqrt{2}\, t\right]$$

```
e6 = e5 /. (C[1] + C[3]) → c1
```

$$c1 + (C[1] - C[3])\, \text{Cos}\left[\sqrt{2}\, t\right] + \sqrt{2}\, C[2]\, \text{Sin}\left[\sqrt{2}\, t\right]$$

```
e7 = e6 /. (C[1] - C[3]) → -c2
```

$$c1 - c2\, \text{Cos}\left[\sqrt{2}\, t\right] + \sqrt{2}\, C[2]\, \text{Sin}\left[\sqrt{2}\, t\right]$$

$$\text{e8} = \text{e7} \;/.\; \left(\sqrt{2}\, C[2]\right) \to c3$$

$$c1 - c2\, \text{Cos}\left[\sqrt{2}\, t\right] + c3\, \text{Sin}\left[\sqrt{2}\, t\right]$$

```
e9 = e2[[1, 2, 2, 2]]
```

$$C[2]\, \text{Cos}\left[\sqrt{2}\, t\right] - \frac{C[1]\, \text{Sin}\left[\sqrt{2}\, t\right]}{\sqrt{2}} + \frac{C[3]\, \text{Sin}\left[\sqrt{2}\, t\right]}{\sqrt{2}}$$

```
e10 = Expand[2 e9]
```

$2\,C[2]\,\text{Cos}[\sqrt{2}\,t] - \sqrt{2}\,C[1]\,\text{Sin}[\sqrt{2}\,t] + \sqrt{2}\,C[3]\,\text{Sin}[\sqrt{2}\,t]$

$e11 = e10\ /.\ C[2] \to \frac{c3}{\sqrt{2}}$

$\sqrt{2}\,c3\,\text{Cos}[\sqrt{2}\,t] - \sqrt{2}\,C[1]\,\text{Sin}[\sqrt{2}\,t] + \sqrt{2}\,C[3]\,\text{Sin}[\sqrt{2}\,t]$

$e12 = \text{Collect}[e11, \sqrt{2}\,\text{Sin}[\sqrt{2}\,t]]$

$\sqrt{2}\,c3\,\text{Cos}[\sqrt{2}\,t] + \sqrt{2}\,(-C[1] + C[3])\,\text{Sin}[\sqrt{2}\,t]$

```
e13 = e12 /. (-C[1] + C[3]) → c2
```

$\sqrt{2}\,c3\,\text{Cos}[\sqrt{2}\,t] + \sqrt{2}\,c2\,\text{Sin}[\sqrt{2}\,t]$

```
e14 = e2[[1, 3, 2, 2]]
```

$\frac{1}{2}C[1]\left(1 - \text{Cos}[\sqrt{2}\,t]\right) + \frac{1}{2}C[3]\left(1 + \text{Cos}[\sqrt{2}\,t]\right) - \frac{C[2]\,\text{Sin}[\sqrt{2}\,t]}{\sqrt{2}}$

```
e15 = Expand[2 e14]
```

$C[1] + C[3] - C[1]\,\text{Cos}[\sqrt{2}\,t] + C[3]\,\text{Cos}[\sqrt{2}\,t] - \sqrt{2}\,C[2]\,\text{Sin}[\sqrt{2}\,t]$

```
e16 = e15 /. (C[1] + C[3]) → c1
```

$c1 - C[1]\,\text{Cos}[\sqrt{2}\,t] + C[3]\,\text{Cos}[\sqrt{2}\,t] - \sqrt{2}\,C[2]\,\text{Sin}[\sqrt{2}\,t]$

$e17 = \text{Collect}[e16, \text{Cos}[\sqrt{2}\,t]]$

$c1 + (-C[1] + C[3])\,\text{Cos}[\sqrt{2}\,t] - \sqrt{2}\,C[2]\,\text{Sin}[\sqrt{2}\,t]$

```
e18 = e17 /. (-C[1] + C[3]) → c2
```

$c1 + c2\,\text{Cos}[\sqrt{2}\,t] - \sqrt{2}\,C[2]\,\text{Sin}[\sqrt{2}\,t]$

$e19 = e18\ /.\ \left(\sqrt{2}\,C[2]\right) \to c3$

$c1 + c2\,\text{Cos}[\sqrt{2}\,t] - c3\,\text{Sin}[\sqrt{2}\,t]$

1. Above: The function forms for y1, y2, and y3 match the green cells above in order, conforming to the text function forms. The system of symbolic constant conversion established

for y1 was carried forward and used for the remaining two functions. It was found that this system matched the text symbolic constant assignments exactly. Each function was upscaled by 2 during assembly.

9. $y_1' = 10 y_1 - 10 y_2 - 4 y_3$
$y_2' = -10 y_1 + y_2 - 14 y_3$
$y_3' = -4 y_1 - 14 y_2 - 2 y_3$

```
ClearAll["Global`*"]

e1 = {y1'[t] == 10 y1[t] - 10 y2[t] - 4 y3[t],
  y2'[t] == -10 y1[t] + y2[t] - 14 y3[t],
  y3'[t] == -4 y1[t] - 14 y2[t] - 2 y3[t]}
e2 = DSolve[e1, {y1, y2, y3}, t]
```

{y1′[t] == 10 y1[t] - 10 y2[t] - 4 y3[t],
 y2′[t] == -10 y1[t] + y2[t] - 14 y3[t], y3′[t] == -4 y1[t] - 14 y2[t] - 2 y3[t]}

{{y1 → Function[{t}, $\frac{1}{9} e^{-18 t} \left(1 + 4 e^{27 t} + 4 e^{36 t}\right) C[1] - \frac{2}{9} e^{-18 t} \left(-1 - e^{27 t} + 2 e^{36 t}\right) C[2] + \frac{2}{9} e^{-18 t} \left(1 - 2 e^{27 t} + e^{36 t}\right) C[3]$],
 y2 → Function[{t}, $-\frac{2}{9} e^{-18 t} \left(-1 - e^{27 t} + 2 e^{36 t}\right) C[1] + \frac{1}{9} e^{-18 t} \left(4 + e^{27 t} + 4 e^{36 t}\right) C[2] - \frac{2}{9} e^{-18 t} \left(-2 + e^{27 t} + e^{36 t}\right) C[3]$],
 y3 → Function[{t}, $\frac{2}{9} e^{-18 t} \left(1 - 2 e^{27 t} + e^{36 t}\right) C[1] - \frac{2}{9} e^{-18 t} \left(-2 + e^{27 t} + e^{36 t}\right) C[2] + \frac{1}{9} e^{-18 t} \left(4 + 4 e^{27 t} + e^{36 t}\right) C[3]$]}}

```
e3 = e2[[1, 1, 2, 2]]
```

$$\frac{1}{9} e^{-18 t} \left(1 + 4 e^{27 t} + 4 e^{36 t}\right) C[1] - \frac{2}{9} e^{-18 t} \left(-1 - e^{27 t} + 2 e^{36 t}\right) C[2] + \frac{2}{9} e^{-18 t} \left(1 - 2 e^{27 t} + e^{36 t}\right) C[3]$$

```
Expand[e3]
```

$$\frac{1}{9} e^{-18 t} C[1] + \frac{4}{9} e^{9 t} C[1] + \frac{4}{9} e^{18 t} C[1] + \frac{2}{9} e^{-18 t} C[2] + \frac{2}{9} e^{9 t} C[2] - \frac{4}{9} e^{18 t} C[2] + \frac{2}{9} e^{-18 t} C[3] - \frac{4}{9} e^{9 t} C[3] + \frac{2}{9} e^{18 t} C[3]$$

```
e4 = Expand[9 e3]
```

$$e^{-18 t} C[1] + 4 e^{9 t} C[1] + 4 e^{18 t} C[1] + 2 e^{-18 t} C[2] + 2 e^{9 t} C[2] - 4 e^{18 t} C[2] + 2 e^{-18 t} C[3] - 4 e^{9 t} C[3] + 2 e^{18 t} C[3]$$

```
e5 = Collect[e4, e^(-18 t)]
```

$e^{9t}(4C[1]+2C[2]-4C[3]) + e^{18t}(4C[1]-4C[2]+2C[3]) + e^{-18t}(C[1]+2C[2]+2C[3])$

```
e6 = e5 /. (C[1] + 2 C[2] + 2 C[3]) → 1/2 c1
```

$\frac{1}{2}c1\, e^{-18t} + e^{9t}(4C[1]+2C[2]-4C[3]) + e^{18t}(4C[1]-4C[2]+2C[3])$

```
e7 = e6 /. (4 C[1] + 2 C[2] - 4 C[3]) → 2 c2
```

$\frac{1}{2}c1\, e^{-18t} + 2\,c2\, e^{9t} + e^{18t}(4C[1]-4C[2]+2C[3])$

```
e8 = e7 /. (4 C[1] - 4 C[2] + 2 C[3]) → -c3
```

$\frac{1}{2}c1\, e^{-18t} + 2\,c2\, e^{9t} - c3\, e^{18t}$

```
e9 = e2[[1, 2, 2, 2]]
```

$-\frac{2}{9}e^{-18t}\left(-1-e^{27t}+2e^{36t}\right)C[1] + \frac{1}{9}e^{-18t}\left(4+e^{27t}+4e^{36t}\right)C[2] - \frac{2}{9}e^{-18t}\left(-2+e^{27t}+e^{36t}\right)C[3]$

```
e10 = Expand[9 e9]
```

$2e^{-18t}C[1] + 2e^{9t}C[1] - 4e^{18t}C[1] + 4e^{-18t}C[2] + e^{9t}C[2] + 4e^{18t}C[2] + 4e^{-18t}C[3] - 2e^{9t}C[3] - 2e^{18t}C[3]$

```
e11 = Collect[e10, e^(-18 t)]
```

$e^{9t}(2C[1]+C[2]-2C[3]) + e^{18t}(-4C[1]+4C[2]-2C[3]) + e^{-18t}(2C[1]+4C[2]+4C[3])$

```
e12 = e11 /. (2 C[1] + C[2] - 2 C[3]) → c2
```

$c2\, e^{9t} + e^{18t}(-4C[1]+4C[2]-2C[3]) + e^{-18t}(2C[1]+4C[2]+4C[3])$

```
e13 = e12 /. (-4 C[1] + 4 C[2] - 2 C[3]) → c3
```

$c2\, e^{9t} + c3\, e^{18t} + e^{-18t}(2C[1]+4C[2]+4C[3])$

```
e14 = e13 /. (2 C[1] + 4 C[2] + 4 C[3]) → c1
```

$$c1\, e^{-18\,t} + c2\, e^{9\,t} + c3\, e^{18\,t}$$

```
e15 = e2[[1, 2, 2, 2]]
```

$$-\frac{2}{9} e^{-18\,t} \left(-1 - e^{27\,t} + 2\, e^{36\,t}\right) C[1] + \frac{1}{9} e^{-18\,t} \left(4 + e^{27\,t} + 4\, e^{36\,t}\right) C[2] - \frac{2}{9} e^{-18\,t} \left(-2 + e^{27\,t} + e^{36\,t}\right) C[3]$$

```
e16 = Expand[9 e15]
```

$$2\, e^{-18\,t} C[1] + 2\, e^{9\,t} C[1] - 4\, e^{18\,t} C[1] + 4\, e^{-18\,t} C[2] + e^{9\,t} C[2] + 4\, e^{18\,t} C[2] + 4\, e^{-18\,t} C[3] - 2\, e^{9\,t} C[3] - 2\, e^{18\,t} C[3]$$

$$\text{e17} = \text{Collect}\left[\text{e16},\, e^{-18\,t}\right]$$

$$e^{9\,t} (2\, C[1] + C[2] - 2\, C[3]) + e^{18\,t} (-4\, C[1] + 4\, C[2] - 2\, C[3]) + e^{-18\,t} (2\, C[1] + 4\, C[2] + 4\, C[3])$$

```
e18 = e17 /. (2 C[1] + C[2] - 2 C[3]) → -2 c2
```

$$-2\, c2\, e^{9\,t} + e^{18\,t} (-4\, C[1] + 4\, C[2] - 2\, C[3]) + e^{-18\,t} (2\, C[1] + 4\, C[2] + 4\, C[3])$$

$$\text{e19} = \text{e18} \;/.\; (-4\, C[1] + 4\, C[2] - 2\, C[3]) \to -\frac{1}{2} c3$$

$$-2\, c2\, e^{9\,t} - \frac{1}{2} c3\, e^{18\,t} + e^{-18\,t} (2\, C[1] + 4\, C[2] + 4\, C[3])$$

```
e20 = e19 /. (2 C[1] + 4 C[2] + 4 C[3]) → c1
```

$$c1\, e^{-18\,t} - 2\, c2\, e^{9\,t} - \frac{1}{2} c3\, e^{18\,t}$$

$$\text{Solve}\left[(2\, C[1] + 4\, C[2] + 4\, C[3]) == c1\ \&\&\ (-4\, C[1] + 4\, C[2] - 2\, C[3]) == -\frac{1}{2} c3\ \&\&\ (2\, C[1] + C[2] - 2\, C[3]) == -2\, c2,\ \{c1, c2, c3\}\right]$$

$$\left\{\left\{c1 \to 2\, (C[1] + 2\, C[2] + 2\, C[3]),\ c2 \to \frac{1}{2} (-2\, C[1] - C[2] + 2\, C[3]),\ c3 \to 4\, (2\, C[1] - 2\, C[2] + C[3])\right\}\right\}$$

```
Solve[(2 C[1] + 4 C[2] + 4 C[3]) == c1 && (-4 C[1] + 4 C[2] - 2 C[3]) == c3 &&
  (2 C[1] + C[2] - 2 C[3]) == c2, {c1, c2, c3}]
```

```
{{c1 → 2 (C[1] + 2 C[2] + 2 C[3]),
  c2 → 2 C[1] + C[2] - 2 C[3], c3 → -2 (2 C[1] - 2 C[2] + C[3])}}
```

```
Solve[(4 C[1] - 4 C[2] + 2 C[3]) == -c3 && (4 C[1] + 2 C[2] - 4 C[3]) == 2 c2 &&
  (C[1] + 2 C[2] + 2 C[3]) == 1/2 c1, {c1, c2, c3}]
```

```
{{c1 → 2 (C[1] + 2 C[2] + 2 C[3]),
  c2 → 2 C[1] + C[2] - 2 C[3], c3 → -2 (2 C[1] - 2 C[2] + C[3])}}
```

1. Above: Referring to green cells top to bottom, the function expressions match those of the text, y1, y2, y3, respectively. The constant coefficients were substituted as required to match those of the text; the three Solve jobs just above (pink cells) verify the coefficient assignment system as consistent and equivalent to the text.

10 - 15 IVPs
Solve the following initial value problems.

11. $y_1' = 2 y_1 + 5 y_2$

$y_2' = -\frac{1}{2} y_1 - \frac{3}{2} y_2$

$y_1[0] = -12, \ y_2[0] = 0$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == 2 y1[t] + 5 y2[t],
  y2'[t] == -1/2 y1[t] - 3/2 y2[t], y1[0] == -12, y2[0] == 0}
e2 = DSolve[e1, {y1, y2}, t]
```

$\{y1'[t] == 2\, y1[t] + 5\, y2[t],\ y2'[t] == -\frac{y1[t]}{2} - \frac{3\, y2[t]}{2},\ y1[0] == -12,\ y2[0] == 0\}$

$\{\{y1 \to \text{Function}[\{t\}, -4\, e^{-t/2} (-2 + 5\, e^{3t/2})],\ y2 \to \text{Function}[\{t\}, 4\, e^{-t/2} (-1 + e^{3t/2})]\}\}$

```
e3 = e2[[1, 1, 2, 2]]
```

$-4\, e^{-t/2} (-2 + 5\, e^{3t/2})$

```
e4 = Expand[e3]
```

$8\, e^{-t/2} - 20\, e^{t}$

```
e5 = e2[[1, 2, 2, 2]]
```

$4\, e^{-t/2} \left(-1 + e^{3\,t/2}\right)$

```
e6 = Expand[e5]
```

$-4\, e^{-t/2} + 4\, e^{t}$

1. Above: The expressions match the text answer for y1 (top green cell) and y2 (bottom green cell).

13. $y_1' = y_2$
$y_2' = y_1$
$y_1[0] = 0,\ y_2[0] = 2$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == y2[t], y2'[t] == y1[t], y1[0] == 0, y2[0] == 2}
e2 = DSolve[e1, {y1, y2}, t]
```

{y1′[t] == y2[t], y2′[t] == y1[t], y1[0] == 0, y2[0] == 2}

$\left\{\left\{y1 \to \text{Function}\left[\{t\}, e^{-t}\left(-1 + e^{2\,t}\right)\right], y2 \to \text{Function}\left[\{t\}, e^{-t}\left(1 + e^{2\,t}\right)\right]\right\}\right\}$

```
e3 = e2[[1, 1, 2, 2]]
```

$e^{-t}\left(-1 + e^{2\,t}\right)$

```
e4 = ExpToTrig[e3]
```

(Cosh[t] - Sinh[t]) (-1 + Cosh[2 t] + Sinh[2 t])

```
e5 = Expand[e4]
```

-Cosh[t] + Cosh[t] Cosh[2 t] + Sinh[t] -
Cosh[2 t] Sinh[t] + Cosh[t] Sinh[2 t] - Sinh[t] Sinh[2 t]

```
e6 = Simplify[e5]
```

2 Sinh[t]

```
e7 = e2[[1, 2, 2, 2]]
```

$e^{-t}\left(1 + e^{2\,t}\right)$

```
e8 = ExpToTrig[e7]
```

(Cosh[t] - Sinh[t]) (1 + Cosh[2 t] + Sinh[2 t])

```
e9 = Expand[e8]
```

Cosh[t] + Cosh[t] Cosh[2 t] - Sinh[t] -
Cosh[2 t] Sinh[t] + Cosh[t] Sinh[2 t] - Sinh[t] Sinh[2 t]

```
e10 = Simplify[e9]
```

```
2 Cosh[t]
```

1. Above: The expressions for y1 (top green) and y2 (bottom green) match the text.

15. $y_1' = 3y_1 + 2y_2$
$y_2' = 2y_1 + 3y_2$
$y_1[0] = 0.5,\ y_2[0] = -0.5$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == 3 y1[t] + 2 y2[t],
  y2'[t] == 2 y1[t] + 3 y2[t], y1[0] == 0.5, y2[0] == -0.5}
e2 = DSolve[e1, {y1, y2}, t]
```

```
{y1′[t] == 3 y1[t] + 2 y2[t],
 y2′[t] == 2 y1[t] + 3 y2[t], y1[0] == 0.5, y2[0] == -0.5}
```

$\{\{y1 \to \text{Function}[\{t\}, 0.5\, e^t], y2 \to \text{Function}[\{t\}, -0.5\, e^t]\}\}$

```
Simplify[e1 /. e2]
{{True, True, True, True}}
```

1. Above: The expression for y1 matches the text. The expression for y2 differs by sign. However, I think the text may be wrong in this case, as the Mathematica answer checks, (and the text's repetitiion of the same answer on two consecutive lines looks funny).

16 - 17 Conversion
Find a general solution by conversion to a single ODE.

17. The system of example 5, p. 144 of the text. That would be conversion of

$$y = c_1 \begin{pmatrix} 1 \\ i \end{pmatrix} e^{(-1+i)t} + c_2 \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{(-1-i)t}$$

into a real general solution by the Euler formula.

19. Network. Show that a model for the currents $I_1(t)$ and $I_2(t)$ in the figure below,

$$\frac{1}{c}\int I_1\, dt + R\,(I_1 - I_2) = 0,\quad L I_2' + R\,(I_2 - I_1) = 0.$$

Find a general solution, assuming that R=3Ω, L=4 H, C=$\frac{1}{12}$F.

Out[52]=